Frequency-Domain System Identification for Linear Time-Periodic Systems with Application to Wind Turbine Dynamics and CSLDV

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Experimental Studies of Dynamic Systems

- Why perform tests?
  - Model Validation
    - "All models are wrong" (D. Smallwood)
  - Exploratory Tests
    - Diagnose Failure
    - Health Monitoring
    - Some physics remain poorly understood; tests are needed to help to create models and guide design.
      - Aerodynamics
      - Coupling between structure and nonlinear magnetic generator?
      - Backlash in gears?
- How? …

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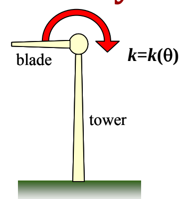
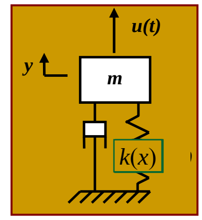
Some Models for Dynamic Systems

- Nonlinear State Space

$$\boxed{\begin{aligned}\dot{x} &= \mathbf{f}(x,t,u) \\ y &= \mathbf{g}(x,t,u)\end{aligned}}$$

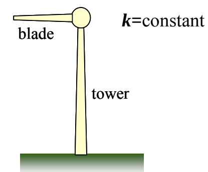




- Linear Time Invariant (LTI)
  - Appropriate for structures with linear force-displacement relationships

$$\boxed{\begin{aligned}\dot{x} &= \mathbf{A}x + \mathbf{B}u \\ y &= \mathbf{C}x + \mathbf{D}u\end{aligned}}$$

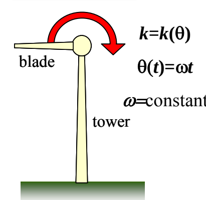
linearize about a single state



- Linear Time Periodic (LTP)
  - Any nonlinear system linearized about a periodic orbit. Common for rotating structures.

$$\boxed{\begin{aligned}\dot{x} &= \mathbf{A}(t)x + \mathbf{B}(t)u \\ y &= \mathbf{C}(t)x + \mathbf{D}(t)u\end{aligned}}$$

linearize about a periodic motion



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Experimental Methods Available

- Nonlinear State Space

$$\boxed{\begin{aligned}\dot{x} &= \mathbf{f}(x,t,u) \\ y &= \mathbf{g}(x,t,u)\end{aligned}}$$

Very limited beyond 1st or 2nd order!

- Linear Time Invariant (LTI)
  - Appropriate for structures with linear force-displacement relationships

$$\boxed{\begin{aligned}\dot{x} &= \mathbf{A}x + \mathbf{B}u \\ y &= \mathbf{C}x + \mathbf{D}u\end{aligned}}$$

Well established time and frequency domain methods routinely used up to high system order.

- Linear Time Periodic (LTP)
  - Rotating structures where a parameter changes with time, or any nonlinear system linearized about a periodic orbit.

$$\boxed{\begin{aligned}\dot{x} &= \mathbf{A}(t)x + \mathbf{B}(t)u \\ y &= \mathbf{C}(t)x + \mathbf{D}(t)u\end{aligned}}$$

Allen's research group has recently extended several LTI System Identification Methods to LTP Systems!

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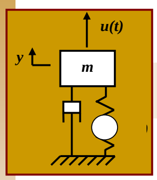
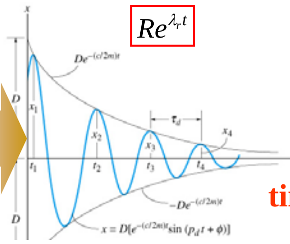
Linear System ID

SDOF

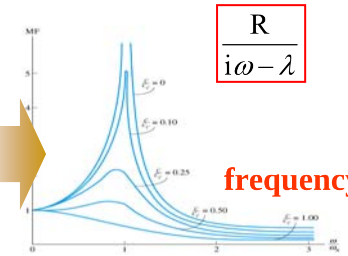


$Re^{\lambda_r t}$



time

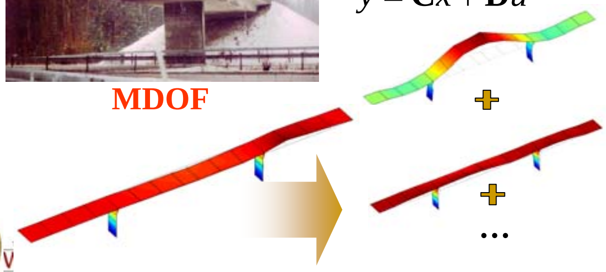
$$\frac{R}{i\omega - \lambda}$$

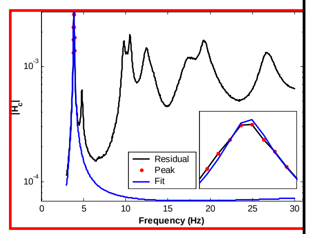


frequency

MDOF

$$\dot{x} = \mathbf{A}x + \mathbf{B}u$$
$$y = \mathbf{C}x + \mathbf{D}u$$

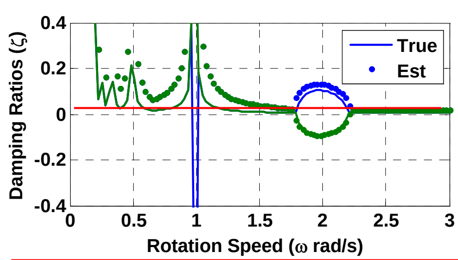




Consequences of Time-Periodic Behavior

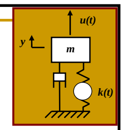
$$\boxed{\begin{aligned}\dot{x} &= \mathbf{A}(t)x + \mathbf{B}(t)u \\ y &= \mathbf{C}(t)x + \mathbf{D}(t)u\end{aligned}}$$

- Parametric resonance
  - Damping depends on the period of the system (e.g. the rotation speed).
  - Damping may be negative (instability) at certain speeds.
- What would be the consequence if a turbine has a parametric resonance that was not accounted for in the design?

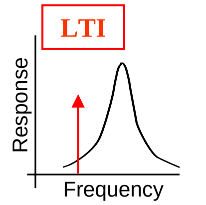


LTP model captures physics that are completely missed by an LTI approximation!!

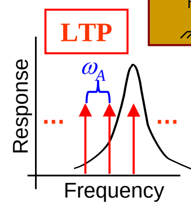
Consequences of Time-Periodic Behavior



- System may respond (i.e. vibrate, generate noise, etc…) at unexpected frequencies.
  - Resonance may be excited by inputs at various frequencies.



$u(t)=u_0\sin(\omega t) \rightarrow y(t)=y_0\sin(\omega t-\phi_0)$

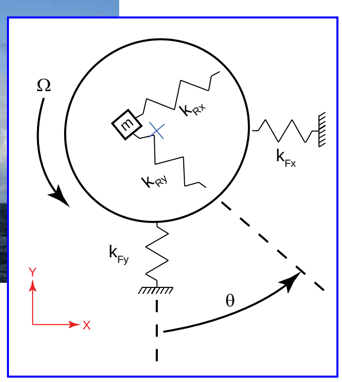


$u(t)=u_0\sin(\omega t) \rightarrow y(t)=y_0\sin(\omega t-\phi_0)$
$+ y_1\sin((\omega+1\omega_A)t-\phi_1)+ \ldots$

Does a wind turbine really need to have three or more blades?

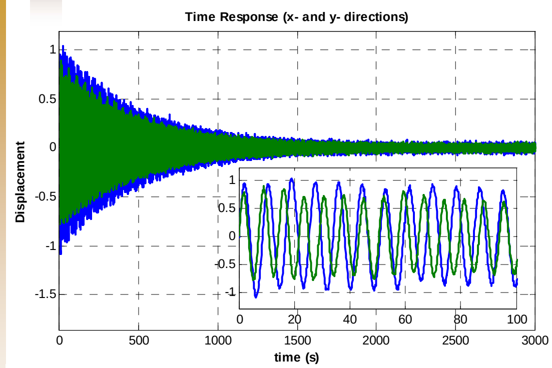
Simulated Identification of Jeffcott Rotor

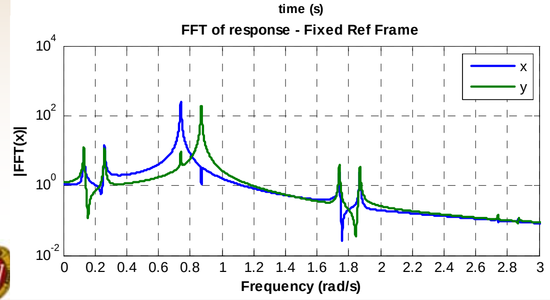
http://en.wikipedia.org/wiki/Wind_turbine



- Jeffcott Rotor on an elastic, anisotropic shaft and an anisotropic foundation.
  - $k_{Rx} = 1$, $k_{Ry} = 1.2$ **20% Stiffer**
  - $k_{Fx} = 1$, $k_{Fy} = 1.5$, **50% Stiffer**
  - **Stability analysis, unstable for:**
    - $0.720 < \Omega < 0.755$
    - $0.785 < \Omega < 0.825$
    - $0.848 < \Omega < 0.920$
- Simulate system identification with constant rotation rate:
  - $\Omega = 0.5$
  - Impulse response simulated in both X and Y directions using time integration.
  - Response sampled 100 times per shaft revolution.

Simulated Measurements





- Impulse response simulated using time integration.
  - "Measured" response contaminated with random measurement noise.
  - Response sampled 100 times per shaft revolution.
- Measurements points at the rotor in both the x- and y- directions (two outputs).
- 2-DOF system responds at eight frequencies?
  - (appears to have 8 modes if thought of as an LTI system)

Identification from Transient Response

- The equations of motion of a general linear time-periodic system can be written as follows with.

$$\dot{\mathrm{x}} = \mathrm{A}(t)\mathrm{x} + \mathrm{B}(t)\mathrm{u}$$
$$\mathrm{y} = \mathrm{C}(t)\mathrm{x} + \mathrm{D}(t)\mathrm{u}$$
$$\mathrm{A}(t) = \mathrm{A}(t+T_A), \text{ etc...}$$
$$\mathrm{x}(t) = \Phi(t,t_0)\mathrm{x}(t_0)$$

- State transition matrix used to develop modal description for LTP system: $\Phi(t,t_0) = \sum_{r=1}^{n} \psi(t)_r L_r(t_0)^{\mathrm{T}} \exp\left(\lambda_r (t - t_0)\right)$
  - Identical to LTI definition except that mode vectors are periodic functions of time:

$$\Psi(t) = \left[\psi(t)_1 \quad \psi(t)_2 \quad \cdots\right], \qquad \left(\Psi(t)^{-1}\right)^{\mathrm{T}} = \left[\mathrm{L}(t)_1 \quad \mathrm{L}(t)_2 \quad \cdots\right]$$

  - The eigenvalues are constant, so each underdamped mode of the STM has a constant natural frequency and damping ratio

$$\Lambda = diag\left[\lambda_1 \quad \lambda_2 \quad \cdots\right] \qquad \lambda_r = -\zeta_r \omega_r + \mathrm{i}\omega_r\sqrt{1-\zeta_r^2}$$

  - These are called the **Floquet exponents** of the LTP system.

Fourier Series Expansion Method

LTP systems have modes that are analogous to the modes of an LTI system.

- LTI Nat. frequencies and damping ratios $\rightarrow$ LTP Floquet Exponents
- LTI Mode shapes (constant) $\rightarrow$ LTP mode shapes (periodic)
    - LTP mode shapes can be shown to give rise to additional frequencies in the response and hence to additional peaks in the spectrum.

$$y(t) = \mathrm{C}(t)\Phi(t,t_0)\mathrm{x}(t_0) \qquad \Phi(t,t_0) = \sum_{r=1}^{n} \psi(t)_r L_r(t_0)^{\mathrm{T}} \exp\left(\lambda_r(t-t_0)\right)$$

- Expand each residue vector in a Fourier series

$$y(t) = \sum_{r=1}^{n} \mathrm{R}_r(t)\exp\left(\lambda_r(t-t_0)\right) \qquad \mathrm{R}_r(t) = \sum_{m=-\infty}^{\infty} \mathrm{R}_{r,m}\exp(im\omega_A t)$$

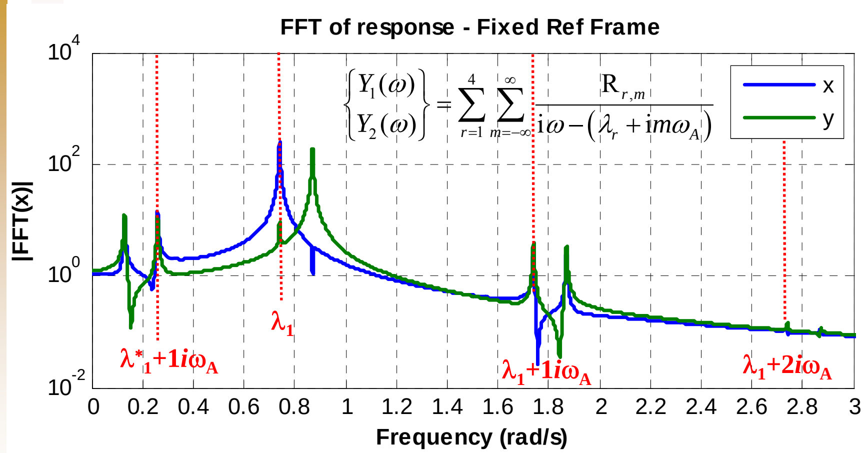
- Insert into eq. for $\mathrm{y}(t)$, setting $\mathrm{t}_0=0$ to simplify the notation.

$$y(t) = \sum_{r=1}^{n}\sum_{m=-\infty}^{\infty} \mathrm{R}_{r,m}\exp\left(\left(\lambda_r + \mathrm{i}m\omega_A\right)t\right)$$

- Or, the FFT can be used to transfer to the frequency domain:

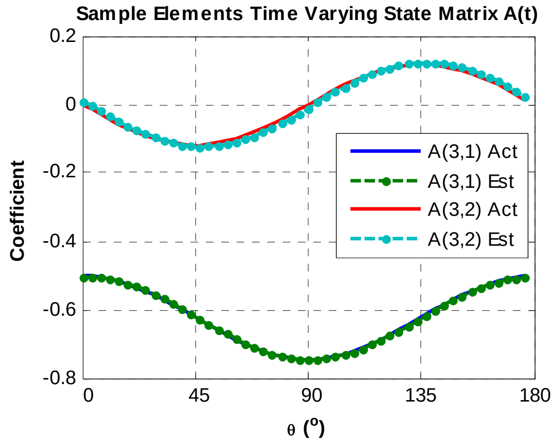
$$y(t) = \sum_{r=1}^{n}\sum_{m=-\infty}^{\infty} \mathrm{R}_{r,m}\exp\left(\left(\lambda_r + \mathrm{i}m\omega_A\right)t\right) \Leftrightarrow Y(\omega) = \sum_{r=1}^{n}\sum_{m=-\infty}^{\infty} \frac{\mathrm{R}_{r,m}}{\mathrm{i}\omega - \left(\lambda_r + \mathrm{i}m\omega_A\right)}$$

Fourier Series Expansion (FSE) of Rotor's Response



- 2-DOF system appears to have 8 modes.
- ω_A=1.0 rad/s (twice the shaft rotation frequency)
- The relationship $\mathrm{Im}\{\lambda_r + \mathrm{i}m\omega_A\}$ can be used to identify which terms $\mathrm{R}_{r,m}$ are present in the Fourier Expansion of $\mathrm{R}_r(t)$.

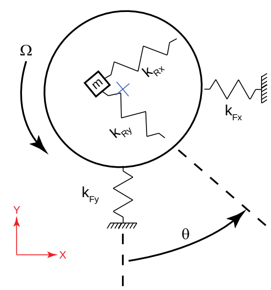
State Matrix



'Est' – estimated, 'Act' – actual

$$\ddot{x} = A(3,1)x + A(3,2)y + \ldots$$

- State matrix A(t) estimated from identified model for state transition matrix.
- Physical interpretation: these give the effective stiffness as a function of shaft angle.
- This information can be used to verify a model, or to predict stability boundaries.



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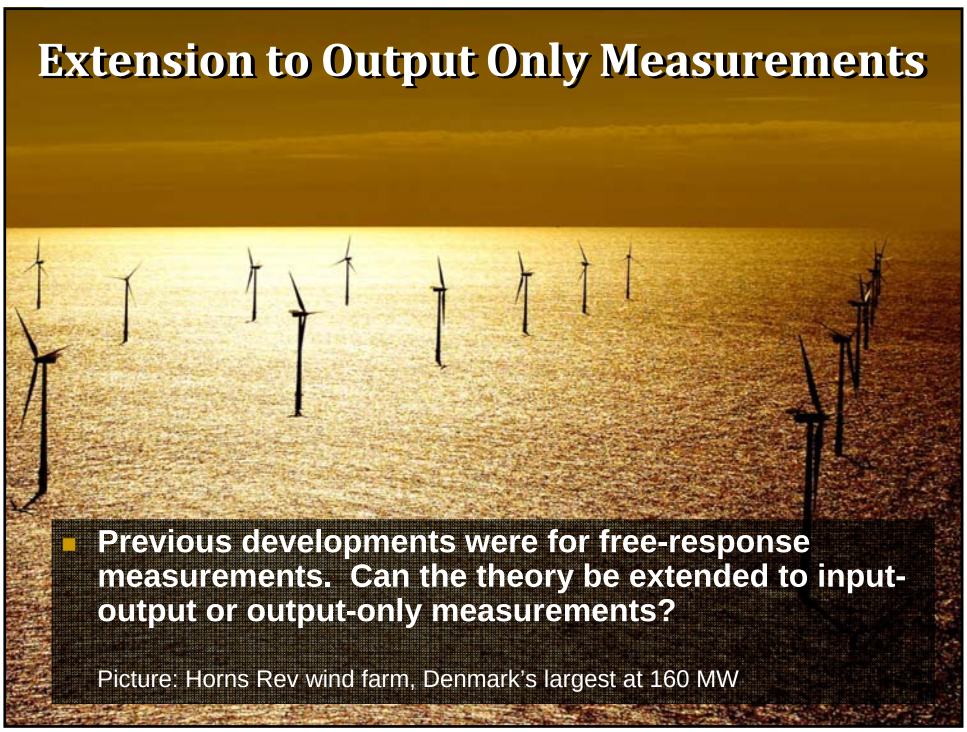
Extension to Output Only Measurements



- **Previous developments were for free-response measurements. Can the theory be extended to input-output or output-only measurements?**

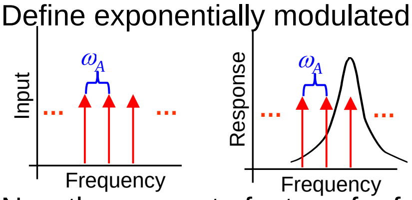
Picture: Horns Rev wind farm, Denmark's largest at 160 MW

Harmonic Transfer Function (HTF)

u(t)=u$_0$sin(ω*t*) → **LTI System** → y(t)=y$_0$sin(ω*t*-γ)

EMP signal → **LTP System** → EMP signal with different magnitude and phase at each frequency

- LTP System: Input at a single frequency causes output at an infinite number of frequencies.
- Define exponentially modulated signal space:



- Now the concept of a transfer function can be readily extended to LTP systems (Wereley, 1991)

$$\mathbf{y}(\omega) = \mathbf{G}(\omega)\mathbf{u}(\omega)$$

N. M. Wereley, PhD Thesis, "Analysis and Control of Linear Periodically Time Varying Systems," *Department of Aeronautics and Astronautics*. Cambridge, MIT, 1991.

HTF: Modal Representation

- As for LTI systems, the HTF can be expressed in terms of the modes:

$$\mathbf{G}(\omega) = \sum_{r=1}^{N}\sum_{l=-\infty}^{\infty} \frac{\bar{\mathbf{C}}_{r,l}\bar{\mathbf{B}}_{r,l}}{i\omega - (\lambda_r - il\omega_A)} + \mathbf{D}$$

$$\bar{\mathbf{C}}_{r,l} = \begin{bmatrix} \cdots & \bar{C}_{r,-1-l}{}^{T} & \bar{C}_{r,-l}{}^{T} & \bar{C}_{r,1-l}{}^{T} & \cdots \end{bmatrix}^T$$

$$\bar{\mathbf{B}}_{r,l} = \begin{bmatrix} \cdots & \bar{B}_{r,l+1} & \bar{B}_{r,l} & \bar{B}_{r,l-1} & \cdots \end{bmatrix}$$

$$C(t)\psi_r(t) = \sum_{n=-\infty}^{\infty} \bar{C}_{r,n} e^{jn\omega_A t}$$

$$L_r(t)^T B(t) = \sum_{n=-\infty}^{\infty} \bar{B}_{r,n} e^{jn\omega_A t}$$

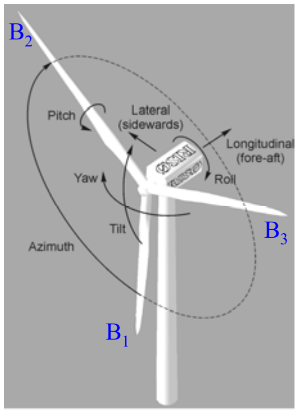
- Output Only Identification based on Autospectrum:

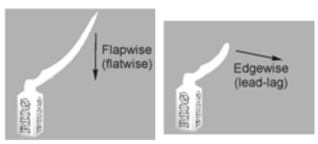
$$\left[S_{yy}(\omega)\right] = \sum_{r=1}^{N}\sum_{l=-\infty}^{\infty} \frac{\bar{\mathbf{C}}_{r,l}\mathbf{W}(\omega)_{r,l}\bar{\mathbf{C}}_{r,l}{}^{H}}{\left[i\omega - (\lambda_r - \mathrm{j}l\omega_A)\right]\left[i\omega - (\lambda_r - \mathrm{j}l\omega_A)\right]^H}$$

Output Autospectrum for an LTI System

$$S_{YY}(\omega) = \sum_{r=1}^{N} \frac{\psi_r S_{UU}(\omega)\psi_r{}^{\mathrm{H}}}{\left[\mathrm{j}\omega - \lambda_r\right]\left[\mathrm{j}\omega - \lambda_r\right]^{\mathrm{H}}}$$

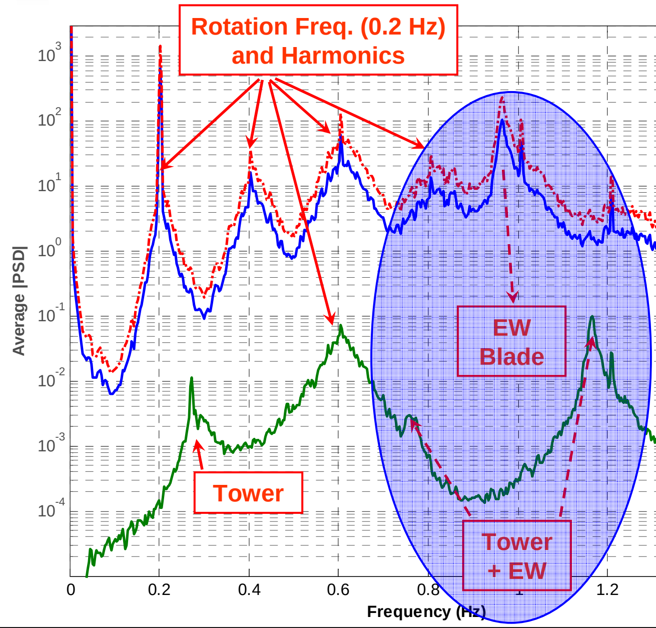
Simulated Application: 5 MW Wind Turbine





- Applied to Simulated Wind Turbine in:
  - M. S. Allen, M. W. Sracic, S. Chauhan, and M. H. Hansen, "Output-Only Modal Analysis of Linear Time Periodic Systems with Application to Wind Turbine Simulation Data," MSSP, vol. 25, pp. 1174-1191, 2011.
- Measurements simulated from the 5MW reference turbine by J. Jonkman:
  - Turbine model created in HAWC2 simulation code (by M. Hansen at RISØ Nat. Lab.)
  - Measurements of turbine simulated for 3.3 hours due to wind excitation.
    - 13.3m turbulence box repeated 16384 times.
    - 75 acceleration measurements simulated on **tower** (three directions) and **blades** (edgewise and flapwise).
- Traditional LTI Modal analysis leads to erroneous results unless a multi-blade coordinate transformation is applied.
  - If the rotor is anisotropic, the system will be LTP even after applying the coordinate transformation.

Traditional Spectra (0 to 1.6 Hz)



Multiple modes seem to exist – explained by LTP theory. Time varying modes identified using harmonic output spectra.

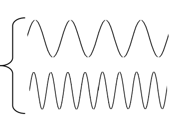
Another Challenge: How to Acquire Measurements?

- Tests difficult due to sheer size
  - Most common sensors must be attached to the structure.
  - Cables must be run from the sensors to data acquisition hardware.
- Laser Doppler Vibrometer:
  - Measures Doppler shift in a beam of laser light → Captures the velocity of the surface at a point.
  - **Advantages:**
    - Non-contact laser measurement simplifies setup
    - Impact excitation is challenging – use the natural excitation from the wind.
  - **Disadvantage**:
    - Captures the response at only one point
    - Too expensive to use many lasers in parallel

CSLDV Solution:

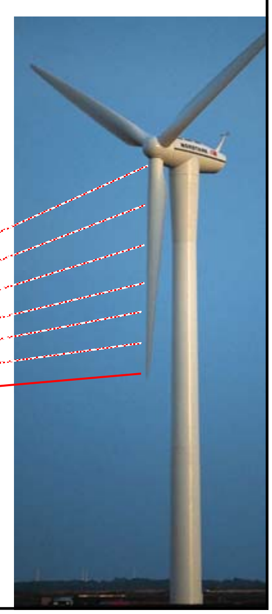
- Continuous-Scan Laser Doppler Vibrometry (CSLDV): Velocity is measured as the laser spot sweeps continuously over the structure.
  - First presented by Sriram & Hanagud (1990)
  - Later extended by Stanbridge, Martarelli & Ewins
    - Sinusoidal Excitation
    - Transient (Impact) Excitation
  - CSLDV with Lifting for Transient Response: Allen & Sracic 2010, Yang, Allen & Sracic 2010

LDV with scanning mirrors



drive signal for scanning mirrors

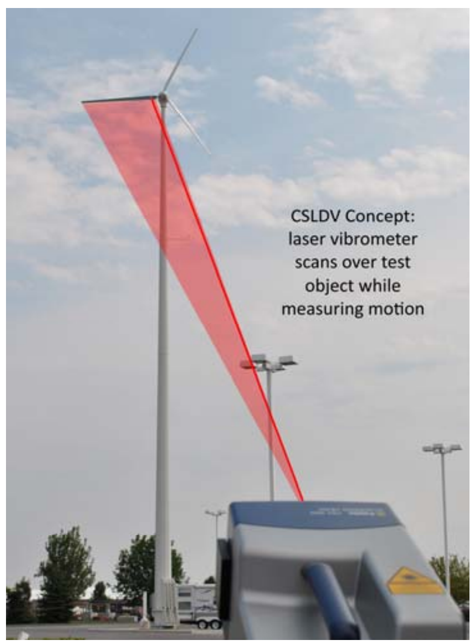
Movie: link 1, link 2



A Useful Laser Show?

OMA-CSLDV on Wind Turbine Blade

- Field Test:
  - Renewegy LLC in Oshkosh, WI: 20kW wind turbine with ~10m diameter rotor, ~30m tower height.
  - Rotor parked (brake applied) during the test.
  - Blade tilted so that the LDV measures in the flapwise direction.
  - The blade was excited by only the ambient wind (3.5 m/s average wind speed) as both conventional and CSLDV measurements were acquired.
  - Retro-reflective tape used, 66.4 meter standoff distance.



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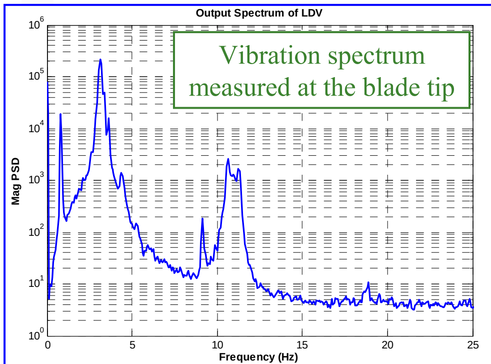
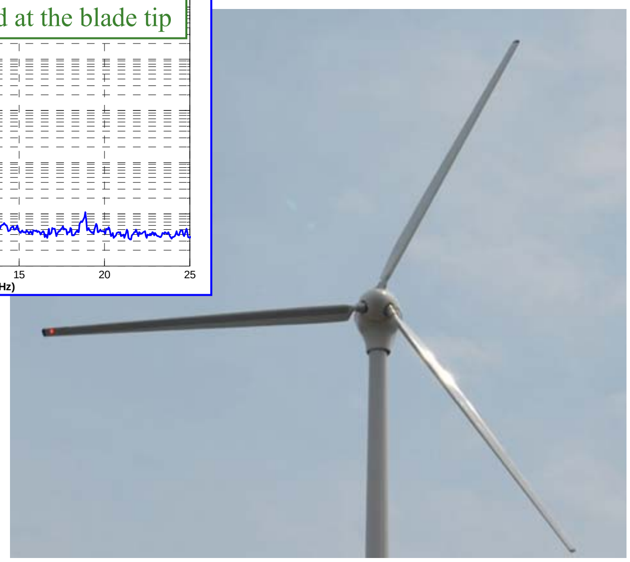
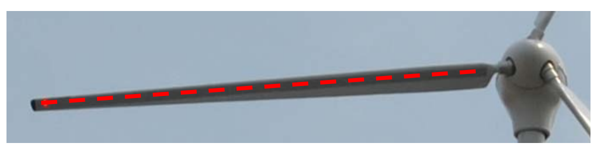
Conventional OMA (Tip Measurement)



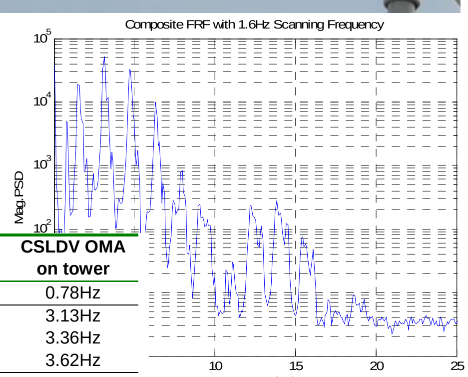


- Laser positioned at a fixed point at the tip of the blade as shown.
- Power spectrum shows several peaks.

OMA-CSLDV Measurement

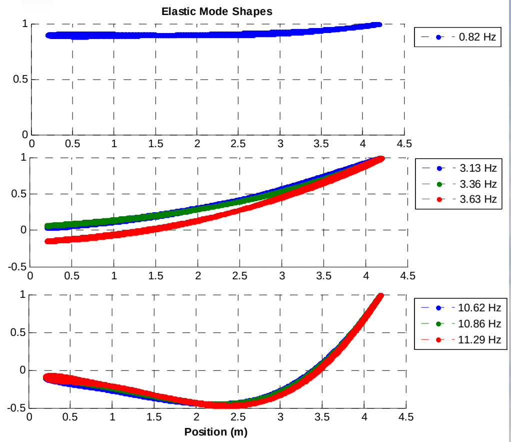


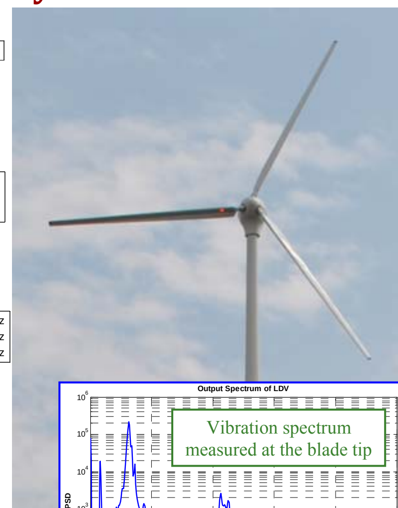
- Laser scanned along the length of the blade for CSLDV measurement.
- Several modes identified in the harmonic power spectrum.

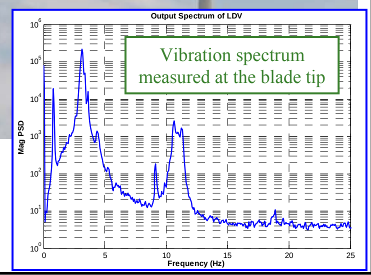


Mode	Conventional test in stiff fixture	Fixed point OMA on tower	CSLDV OMA on tower
-	-	0.81Hz	0.78Hz
Flap Wise Bending 1	3.36Hz	3.13Hz 3.37Hz 3.63Hz	3.13Hz 3.36Hz 3.62Hz
Edge Wise Bending 1	5.24Hz	4.38Hz	-
-	-	9.13Hz	-
Flap Wise Bending 2	11.40Hz	10.63Hz 10.94Hz 11.25Hz	10.62Hz 10.86Hz 11.29Hz

Mode Shapes Identified by OMA CSLDV







- CSLDV reveals the deformation shape of the structure associated with each frequency.

Comparison with Conventional Methods

- Conventional Test Methods:
  - OMA with accelerometers (fixed sensors)
    - Requires attaching sensors to the points of interest and running cables to data acquisition (or wireless transmitters).
- OMA with conventional scanning LDV
  - At least two measurement points needed to obtain mode shapes.
  - Cost per LDV: $80,000+
  - Each pair of points must be observed for at least 10 minutes.
- The results presented here were acquired with one ground based laser and two 10-min time records!

Conclusions & Outlook

- Linear Time Periodic (LTP) systems are capable of modeling many important phenomena.
- Most system identification concepts for linear time-invariant systems extend readily to LTP systems.
  - Frequency Domain Transfer Function
  - Mode Indicator Functions
  - System Identification Routines (for parameter identification)
  - Insight and intuition acquired for LTI systems.
- Outcomes:
  - Continuous-scan Laser Doppler Vibrometry can be used to reduce vibration test time by two orders of magnitude.
  - Many other systems can be treated experimentally using this technique.
    - Rotating machines such as wind turbines, helicopters
    - Nonlinear systems such as the human neuromuscular system...
  - **A short course has been created on these topics and will be presented in upcoming conferences and to industry.**

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Extra Slides

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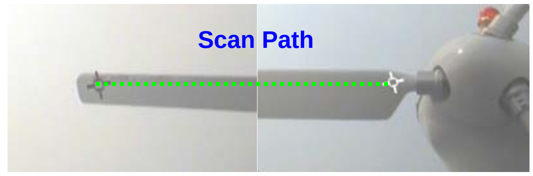
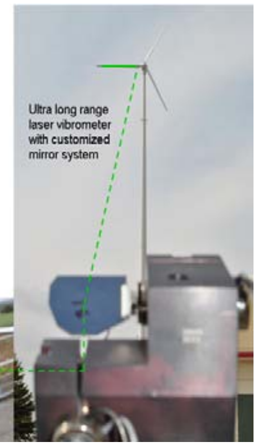
New Remote Sensing Vibrometer

- Field test at Renewegy LLC in Oshkosh, WI:
  - 20kW wind turbine with 9.4m diameter rotor and 30m tower height.
  - 77 m standoff from laser head to target.
  - Rotor was parked (brake applied)
  - The blade was excited by only the ambient wind with 9 m/s max wind speed
- Remote Sensing Vibrometer
  - Prototype from Polytec®
  - 1550nm wavelength
  - Higher power (10mW)
  - Designed for long range measurements, improved signal.



Standoff distance	77m
Blade length	4.5m
Max wind speed	9m/s
Scan Frequency	36Hz



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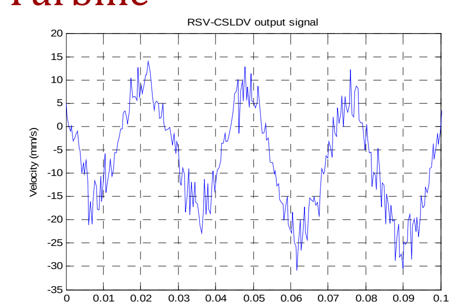
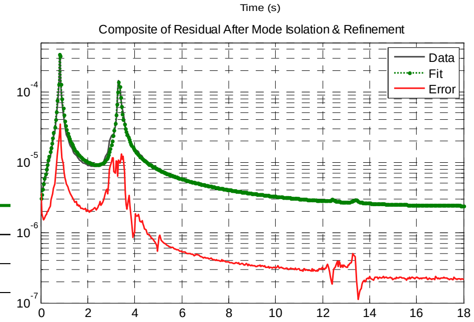
Video of CSLDV with RSV

Application to Wind Turbine

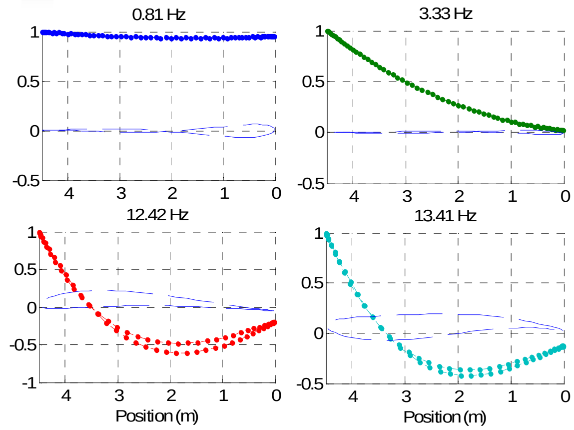
- Retro-reflective tape was not needed for the RSV
  - The conventional 633nm laser required retro-reflective tape
- Surface velocity of the laser spot at a scan frequency of 36Hz is approximately 500m/s.
- 400s time record
- Response dominated by 36Hz but other frequency components are well captured.
  - RSV has great potential for lifting where high scan frequencies are preferred

Mode	Natural frequency	Damping
Tower Bending	0.81Hz	1.61%
Flap Wise Bending 1	3.33Hz	1.52%
Flap Wise Bending 2	12.42Hz 13.41Hz	0.44% 0.70%





Measured Mode Shapes



- Blade moves as a rigid body in the 0.81 Hz mode, revealing that this is a tower bending mode.
- First blade bending mode found to be 3.33Hz
- 12.42Hz and 13.41Hz frequencies revealed to be second blade bending modes
- 400s data allows for 31 averages
  - A longer time history would be preferred.
- Imaginary part of shapes thought to be an artifact of the short data length.
- Noisy second bending mode, weakly excited

Validation with Traditional SLDV

- Laser position determined by measuring the angle of the RSV Laser head
- Measurements agree fairly well with those obtained by CSLDV
  - Two additional peaks could be identified around 4Hz and 5Hz due to lower speckle noise
- There are several points which appear to be questionable
  - Due to changing wind conditions?
- SLDV used 5.3 min long measurements at each of the 5 points = total 26.5min
  - OMA-CSLDV achieves far higher resolution with a single 6.7 min measurement!